Solutions to selected exercises from Chapter 5

5.1

1.

f(x) is quadratic because the second differences are constant.

x	***f(x)***	First differences	Second differences
0	-1.00	0.08	0.16
2	-0.92	0.24	0.16
4	-0.68	0.40	0.16
6	-0.28	0.56	0.16
8	0.28	0.72	0.16
10	1.00	0.88	
12	1.88		

g(x) is cubic because the third differences are constant.

x	***g(x)***	First differences	Second differences	Third differences
0	0.00	-0.88	-1.28	0.72
2	-0.88	-2.16	-0.56	0.72
4	-3.04	-2.72	0.16	0.72
6	-5.76	-2.56	0.88	0.72
8	-8.32	-1.68	1.60	
10	-10.00	-0.08		
12	-10.08			

h(x) is linear because the first differences are constant.

x	***h(x)***	First differences
0	2.50	2.5
2	5.00	2.5
4	7.50	2.5
6	10.00	2.5
8	12.50	2.5
10	15.00	
12	17.50	

6. The concavity changes one time so the type of polynomial function represented by the graph is a cubic function.

9. The concavity does not change so the type of polynomial function represented by the graph is a quadratic function.

12 a. As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$.

b.

x	*f(x)*
0	1
10	941
20	7,881
30	26,821
40	63,761
50	124,701

x	*f(x)*
-50	-124,699
-40	-63,761
-30	-26,819
-20	-7,879
-10	-939
0	1

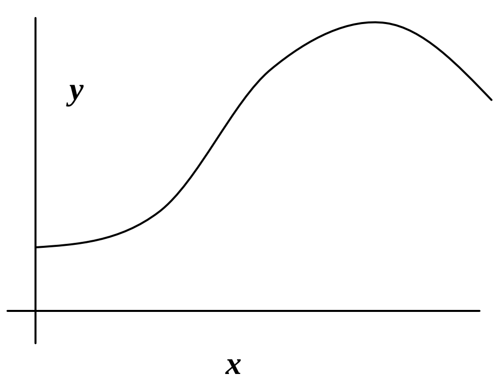
20 a. As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$.

b.

x	*f(x)*
12	497,647
14	2,151,277
16	6,291,435
18	15,116,521
20	31,999,975

x	*f(x)*
0	-5
-2	381
-4	14,335
-6	124,417
-8	589,827

33.



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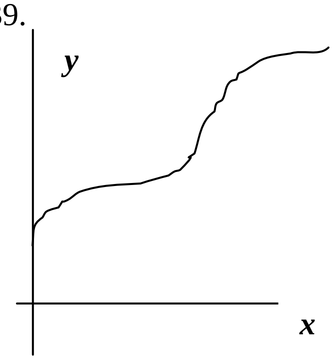

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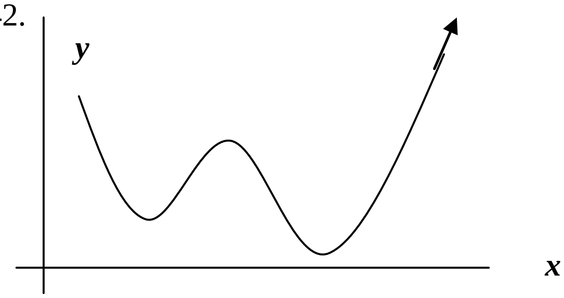

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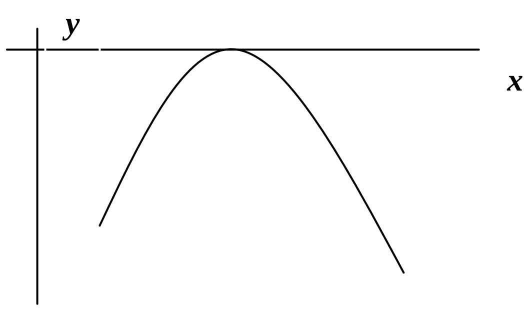
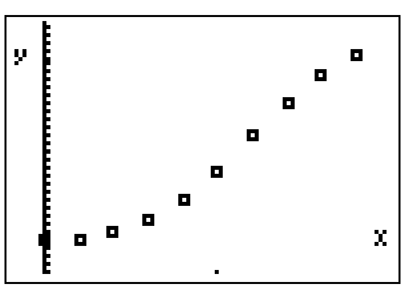


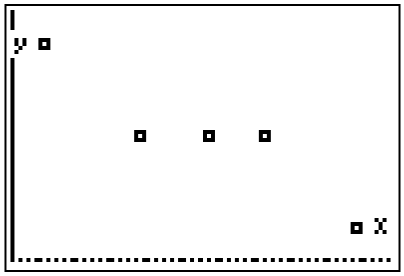
49a. A cubic model best models the data because the concavity changes from concave up to concave down.



b. $R(t) = -0.048t^3 + 0.887t^2 = 1.495t + 58.324$

c. Using the model $R(t)$, the minimum room rate from 1990 to 1999 is approximately in year 1 (1991) at the rate of \$57.66.

53 a. A cubic model best models the data because the concavity changes from concave up to concave down.



b. $S(m) = -0.000917m^3 + 4.21m^2 - 6440.463m + 3{,}320{,}100.975$

c. According to S(m) increasing the number of motion picture screens do not necessarily lead to an increase in movie attendance.

5.3

1a. M(0) ≈ 72. When a male is born (0 years old currently), he is expected to live 72 more years. W(0) ≈ 78. When a female is born (0 years old currently), she is expected to live 78 more years.

b. M(20) ≈ 54. When a male is 20 years old currently, he is expected to live 54 more years. W(20) ≈ 60. When a female is 20 years old currently, she is expected to live 60 more years.

c. The limiting value for both males and females appears to be 0.

d. The horizontal asymptote is W(a) = 0 and M(a)=0. As a person's age increases more and more, the rate of change in expected years to live decreases until it approaches 0. This means that someone very old is not expected to live very much longer.

2a. We know that distance equals the rate times the time. Therefore, 150 miles = 75 miles/hour times the time. Time would equal 2 hours.

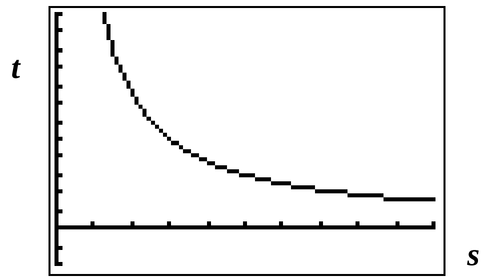
b. $t(s) = \dfrac{150}{s}$

c. As the average speed increases the travel time decreases. This means that if one's average speed increases then the time it takes to complete the journey decreases.

d.

s (average speed)	*t* (hours)
20	7.5
15	10
10	15
5	30
4	37.5
3	50
2	75
1	150

e.



f. The horizontal asymptote is t = 0 which means that as the person's speed increases the time it takes to complete the 150-mile journey becomes shorter and shorter until it can almost reach 0 hours for VERY high speeds. The vertical asymptote is s = 0 which means that as the person's speed approaches very small values (s approaches 0 miles per hour) the time it takes to complete the 150-mile journey becomes very large.

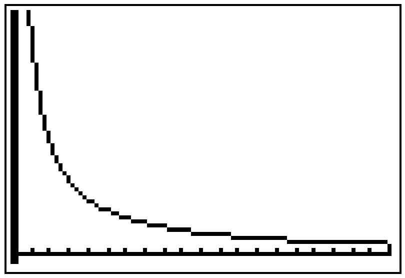
3. a. D(34000, 2000, N) = 32000/N.

b.

N	*D*
1	32000
3	10667
5	6400
7	4571.43
8	3555.56
11	2909.09
13	2461.54
15	2133.33

c. (Answers vary). The practical domain for D(N) is 0 < N < 20.

d.



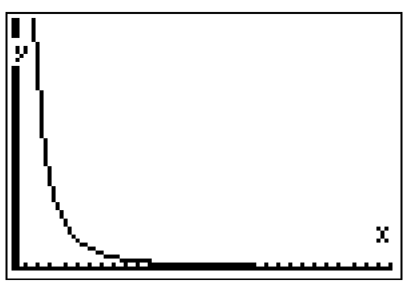
e. The horizontal asymptote is D = 0 which means that as the number of years increase the amount that the car depreciates continues to decrease until the amount of depreciation approaches 0. In other words, the car will always be worth some amount even it if is a very small amount. The vertical asymptote is N = 0 which means that as the number of years approach 0 the depreciation grows quickly. When the car is initially purchased the amount of depreciation is very high.

4 a.

d	***I***
0.1	149,500
0.5	5,980
1	1,495
2	373.75
5	59.8
10	14.95
20	3.7375
30	1.6611

b. (Answers vary) A practical domain would be $0 \le d \le 100$.

c.



d. As you move closer to the person speaking, the sound intensity increases at an increasing rate.

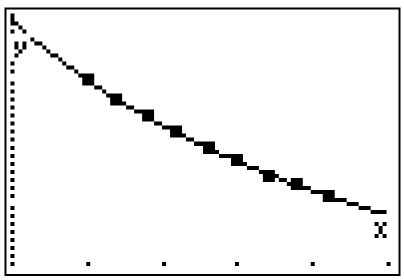
5 a. $B(w,h) = \dfrac{705w}{h^2}$, $w = 190$ pounds, $B(190,h) = \dfrac{705(190)}{h^2} = \dfrac{133,950}{h^2}$

b.

h	***B***
60	37.21
62	34.85
64	32.70
66	30.75
68	28.97
70	27.34
72	25.84
74	24.46
76	23.19

c.

BMI



Height

d. The function decreases at a decreasing rate each increase in height yields a decrease in BMI, but the decreases are less and less.

6 a.

$$0.08 = \frac{600n}{200(169 + 0.6n)}$$
$$16(169 + 0.6n) = 600n$$
$$2704 = 590.4n$$
$$4.56 = n$$

200 pounds can have about 4.5 beers

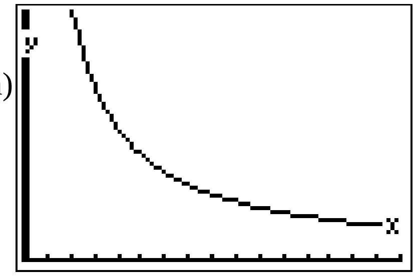
$$0.08 = \frac{600n}{110(169 + 0.6n)}$$
$$8.8(169 + 0.6n) = 600n$$
$$1487.2 = 594.72n$$
$$2.5 = n$$

110 pounds can have about 2.5 beers

b. Yes, a person's weight has an impact on the limit of beers a person can consume legally. The lighter you are the fewer beers can be consumed.

7 a.

Rate (ft/min)



Time (min)

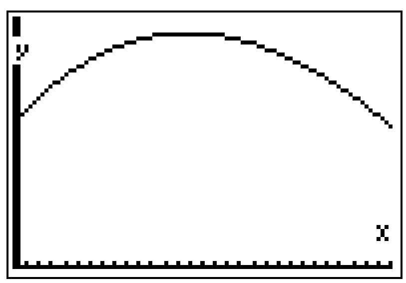
b. Domain: $t \neq 0$

c. Time cannot be negative, nor would it be reasonable to consider very large values of time or very small values of time.

d. As $t \to \infty,\ r(t) \to 0$ that means that if the patient takes “forever” to complete the workout, then the pace of the workout is REALLY slow.

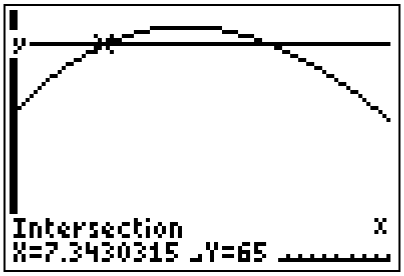
9 a.
$$\begin{aligned} R(t) &= -54t^2 + 560t + 8030 + 19t^2 + 600t + 2860 \\ &= -35t^2 + 1160t + 10890 \end{aligned}$$

b. $A(t) = \dfrac{-35t^2 + 1160t + 10890}{0.0067t^2 + 2.56t + 250.39}$



c.
$$\begin{aligned} A(14) &= \frac{-35(14)^2 + 1160(14) + 10890}{0.0067(14)^2 + 2.56(14) + 250.39} \\ &= \$70.49 \end{aligned}$$

d.



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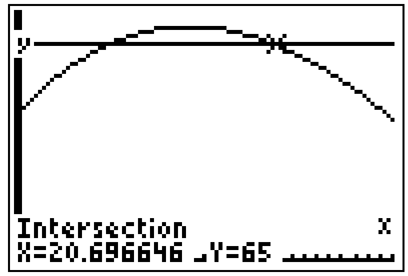


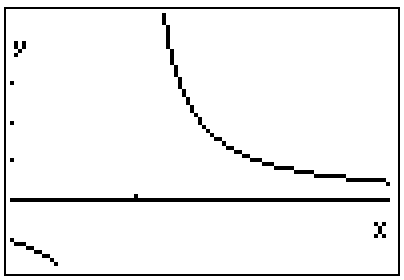
In the years 1997 and 2010.

e. The function is concave down meaning that as the years increase the average annual cost per person increases at a decreasing rate and then decreases at an increasing rate.

10 a. $W(p) = \dfrac{50}{p(p-50)}$, $W(p) =$ wait time in hours

$p =$ average number of people served per hour

People arrive at 50 people per hour



For values just over 50 the wait time is very large because the number of people being served is just barely more than the number of people arriving so the line isn't moving much. But as they serve more people per hour the wait time decreases.

b. The function is meaningless for values less than 50 because if you're serving less than 50 people per hour and the rate that people are arriving is 50 per hour, then you're not making any progress in getting the line down – the wait time is negative.

c. Solving graphically, the approximate number of customers who can be served in 0.5 hours is 52.

d. About 0.5 hours.

e. Solving graphically, for 20 minute wait times the workers must serve approximately 53 people per hour.

11. The horizontal asymptote of $f(x)$ is $f(x)=2$. The vertical asymptote is $x=-4$.

12. The horizontal asymptote of $h(x)$ is $h(x)=0$. The vertical asymptote is x = 3.

13. The horizontal asymptote of $g(x)$ is $g(x)=\frac{-3}{2}$. The are no vertical asymptotes.

14. The horizontal asymptote of $f(x)$ is $f(x)=0$. The vertical asymptote is $x=-\frac{3}{2}$. (Note that $x=1$ is NOT an asymptote but rather a hole.)

15. The x-intercept is (3, 0). The y-intercept is (0, 0.6). The horizontal asymptote is y = 1. There vertical asymptotes is x = 5.

16. The x-intercepts are (-4, 0) and (4,0). The y-intercept is (0, -1). The horizontal asymptote is y = 1. There are no vertical asymptotes.

17. The x-intercepts are (-3, 0) and (3,0). The are no y-intercepts. The horizontal asymptote is y = 0. There vertical asymptotes are x = -9 and x = 0.

18. The x-intercepts are (0, 0) and (2,0). The y-intercept is (0, 0). The horizontal asymptote is y = -1. There vertical asymptotes are x = 1.39 and x = 8.6.

19. The are no x-intercepts. The are y-intercept is (0, -0.5). The horizontal asymptote is y = 0. There vertical asymptote is x = 2.

20.

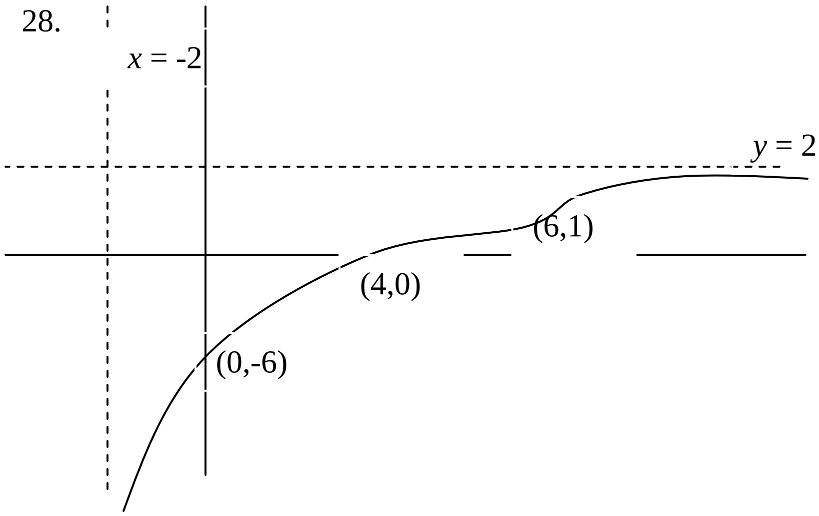
x	-4	-3.1	-3.01	-3	-2.99	-2.9	-2
h(x)	1	100	10000	Undefined	10000	100	1

The values of ***h(x)*** increase without bound as x approaches 3 from the left and right.

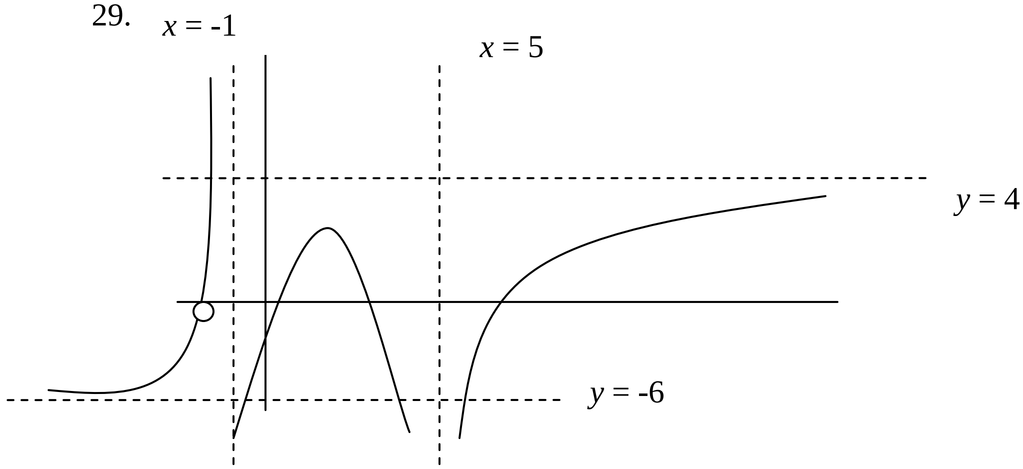
26. The coordinates of the x-intercept is (3.6, 0) and the equation of the vertical asymptote is x = 4.0.

27. y-intercept is (0,0) and the horizontal asymptote is y = 1.

28.



29.



30.

x	***f(x)***
-1	40
0	-2
2	-1
3	-0.5
4	0
6	0.4
8	0.7
9	0.9
20	0.99999

32. (answers vary) $f(x) = \dfrac{4}{(x-6)(x+2)}$

33. (answers vary) $f(x) = \dfrac{-1}{(x-6)^2}$

38. I and II

39. II

43. The table below shows how the values fluctuate between highs and lows, but as $x \to \infty$ $f(x) \to 3$ because the highs and lows get closer and closer to 3.

x	***f(x)***
0	10
5	-4
10	8
15	-1
20	6
25	1
30	5
35	2
40	4